① Elastic & Inelastic Collisions

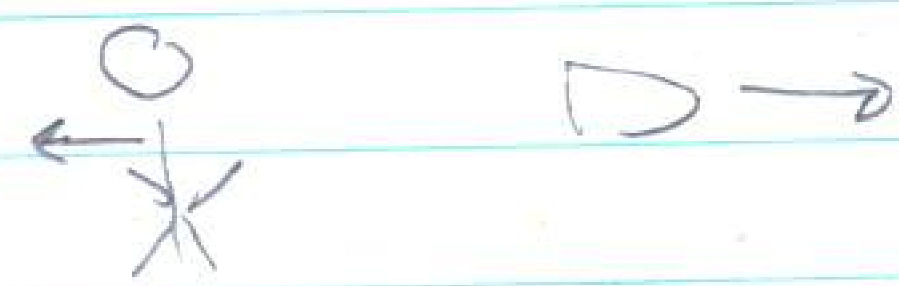
Last Time

① $\vec{p} = m\vec{v}$ is momentum

② Momentum is conserved

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

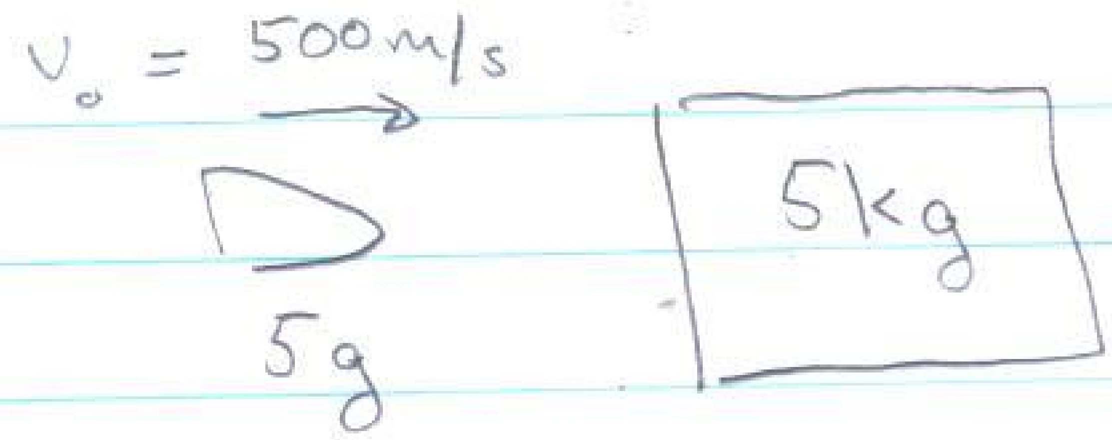
Example: Before   After



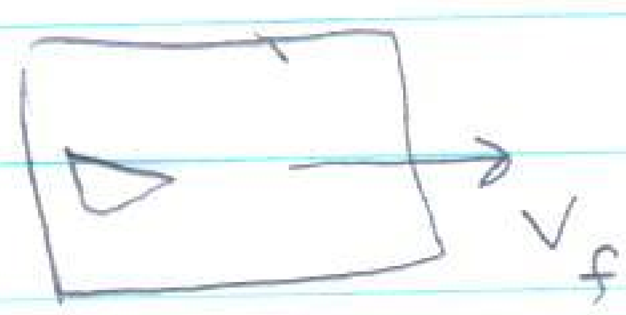
$P_{1i} + P_{2i} = 0$ $\vec{P}_{me} = -\vec{P}_{Bullet}$

Inelastic Collision:

Before



After



- Momentum is conserved
- Energy is usually "lost"

Find V_f:

$$P_{1i} + \cancel{P_{2i}} = P_{1f}$$

$$m_B v_0 = (m_B + M)\, v_f$$

$$\frac{m_B}{m_B + M} V_0 = V_f \qquad V_f = 0.499\,m/s$$

Then If the "collision" took place over a period of 0.01 s what was the average force on the bullet

$$\Delta p = \int_{t_a}^{t_b} \vec{F} \, d_t$$

$$\frac{\Delta p}{\Delta t} = \vec{F}_{ave}$$

$$\Delta p = m_B V_f - m_B V_0$$

$$\frac{\Delta p}{\Delta t} = -m_B \frac{(V_0 - V_f)}{\Delta t} = -(.005\,kg)\frac{(500 - 0.499)\,m/s}{.01\,s}$$

$$= 249.75\,N$$

Elastic Collisions;

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

- Momentum is conserved
- Energy is also conserved

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

So we can simplify in one d

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Goal

$$\underbrace{m_1 (v_{1i} - v_{1f})} = \underbrace{m_2 (v_{2f} - v_{2i})}$$

Given v_{1i} and v_{2i} determine v_{1f} and v_{2f}

In 1D can simplify this:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\frac{1}{2} m_1 (v_{1i}^2 - v_{1f}^2) = \frac{1}{2} m_2 (v_f^2 - v_i^2)$$

$$\frac{1}{2} m_1 \underbrace{(v_{1i} - v_{1f})} (v_{1i} + v_{1f}) = \frac{1}{2} m_2 \underbrace{(v_{2i} - v_{2f})} (v_{2i} + v_{2f})$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$(v_{1i} - v_{2i}) = +(v_{2f} - v_{1f})$$

$$(v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

Example: Before After

$m_1 = 500 g$ $m_2 = 1000 g$

(4)

Solution

$$m_1 v_{1i} + \cancel{m_2 v_{2i}}^{\,0} = m_1 v_{1f} + m_2 v_{2f}$$

Call: $v_{1i} \equiv v_0 \qquad v_{1f} = v_1 \qquad v_{2f} = v_2$

$$m_1 v_0 = m_1 v_1 + m_2 v_2$$

$$(v_{1i} - \cancel{v_{2i}}^{\,0}) = -(v_{1f} - v_{2f})$$

$$v_0 = -v_1 + v_2$$

$$m_1 v_0 = m_1 v_1 + m_2 (v_0 + v_1)$$

$$\left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_0 = v_1 \implies -\left(\frac{m_2 - m_1}{m_2 + m_1}\right) v_0 = v_1 \qquad v_f = 1$$

$$v_0 + v_1 = v_2$$

$$v_0 - \frac{1}{3} v_0 = v_2$$

$$\frac{2}{3} v_0 = v_2$$

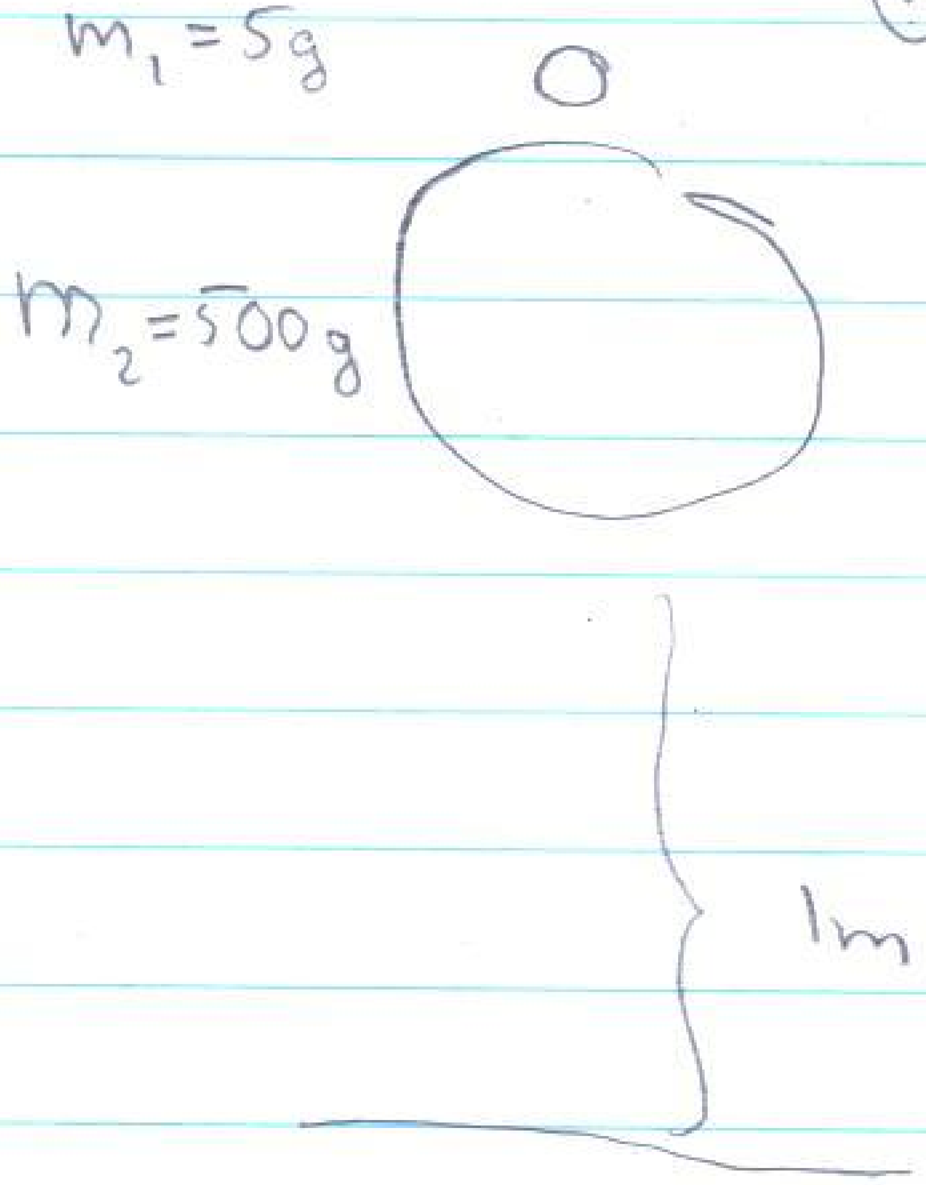
$$2\,m/s = v_2$$

$$-\left(\frac{1000g - 500g}{1500g}\right) v_0 = v_1$$

$$-\frac{1}{3} v_0 = v_1$$

$$-1\,m/s = v_1$$

Example 2 :



1. Find the speed before it hits the ground :

Answer:

$$\cancel{W}_{ext}^{0} = \Delta KE + \Delta PE$$

$$v = \sqrt{2gh}$$

$$v_0 = 4.472 \ m/s$$

2. Before / After

3. Find the final speed of the light ball;

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\rightarrow (v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

(6)

Then $\quad V_{1i} = -V_0 \quad V_{2i} = V_0 \quad V_{1f} = V_1 \quad V_{2f} = V_2$

$$m_1 - V_0 + m_2 V_0 = m_1 V_1 + m_2 V_2$$

$$(-V_0 - V_0) = -(V_1 - V_2)$$

$$2V_0 = (V_1 - V_2)$$

$$-m_1 V_0 + m_2 V_0 = m_1 V_1 + m_2 (V_1 - 2V_0)$$

$$\left(\frac{3m_2 - m_1}{m_1 + m_2}\right) V_0 = V_1$$

$$3 V_0 \approx V_1 \quad \text{for} \quad m_2 \gg m_1$$

$$2.96 \cdot V_0 = V_1$$

$$13.123 \, m/s = V_1$$

(4) Use E-conserve to find the final height

answer: $\quad h = \dfrac{V_1^2}{2g} \approx 9m$

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Summary and 2D

Momentum Conserv,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$ (Inelastic + Elastic)

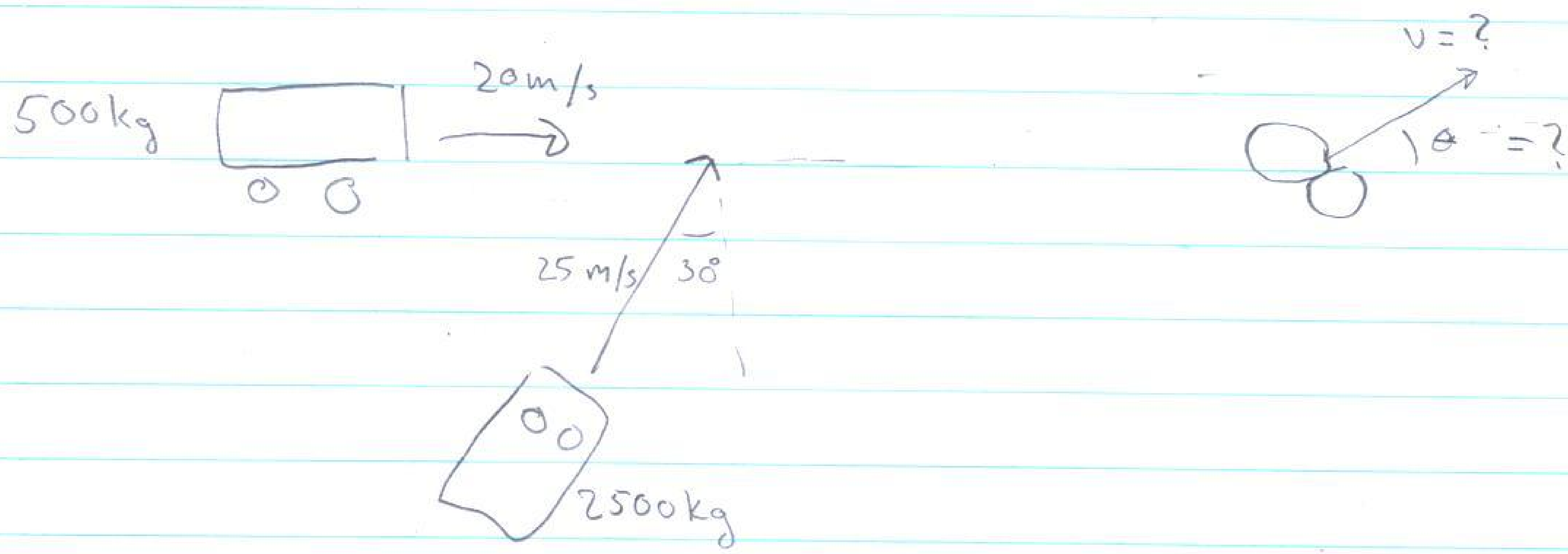
E - Conservation

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$ (Elastic Only)

$$(v_{1f} - v_{2i}) = -(v_{1i} - v_{2i})$$ (Elastic + 1D only)

Example 2D inelastic:

Car collision: Before   After



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Solution

$$\underbrace{m\vec{v}_1 + m_2\vec{v}_2}_{\vec{p}_{\text{initial TOT}}} = \underbrace{(m_1+m_2)\vec{v}}_{\vec{p}_{\text{final TOT}}}$$

$$\vec{v}_1 = \begin{pmatrix} 20\,m/s\,\hat{i} \\ 0 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} 25\,m/s \sin 30^\circ\,\hat{i} \\ 25\,m/s \cos 30^\circ\,\hat{j} \end{pmatrix}$$

$$\vec{v}_2 = \begin{pmatrix} 12.5\,m/s\,\hat{i} \\ 21.65\,m/s\,\hat{j} \end{pmatrix}$$

$$m_1 \begin{pmatrix} 20\,m/s\,\hat{i} \\ 0 \end{pmatrix} + m_2 \begin{pmatrix} 12.5\,m/s\,\hat{i} \\ 21.65\,m/s\,\hat{j} \end{pmatrix} = (m_1+m_2) \begin{pmatrix} V_f^x\,\hat{i} \\ V_f^y\,\hat{j} \end{pmatrix}$$

$$(500\,kg)(20\,m/s) + (2500\,kg)(12.5\,m/s) = (3000\,kg)\,V_f^x$$

$$V_f^x = 13.75\,m/s$$

$$(500\,kg)(0) + (2500\,kg)(21.65\,m/s) = (3000\,kg)\,V_f^y$$

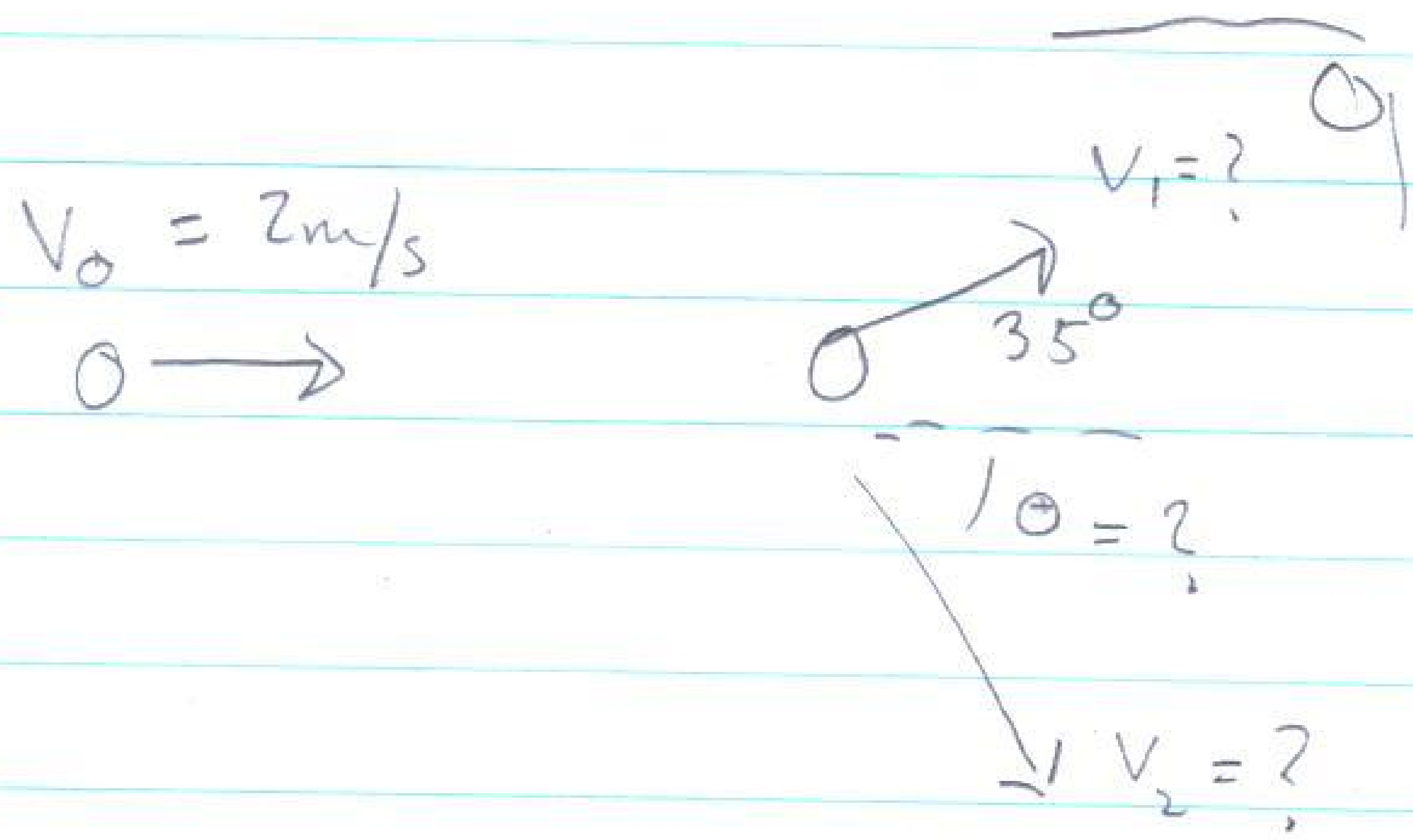
$$18.0\,m/s = V_f^y$$

$$\vec{v} = \begin{pmatrix} 13.75\,m/s \\ 18.0\,m/s \end{pmatrix} \qquad |V| = \sqrt{V_x^2 + V_y^2} = 22.65\,m/s$$

$$\phi = \tan^{-1}\frac{V_y}{V_x} = 52.6^\circ$$

9 2D Elastic

Pool



(9.1) $$m_1\begin{pmatrix}V^x_{1i}\\ \cancelto{0}{V^y_{1i}}\end{pmatrix} + \cancelto{0}{m_2\begin{pmatrix}V^x_{2i}\\ V^y_{2i}\end{pmatrix}} = m_1\begin{pmatrix}V^x_{1f}\\ V^y_{1f}\end{pmatrix} + m_2\begin{pmatrix}V^x_{2f}\\ V^y_{2f}\end{pmatrix}$$

(9.2) $$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 V_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

- All masses equal $m_1 = m_2 = m$

$$V^x_{1i} = v_0$$

So equations become

(9.3) $$\cancel{m} v_0 = \cancel{m_1} V_1 \cos 35^\circ + \cancel{m_2} V_2 \cos\theta$$

(9.4) $$0 = \cancel{m} v_1 \sin 35^\circ - \cancel{m} v_2 \sin\theta$$

(9.5) $$\cancel{\frac{1}{2}m} v_0^2 = \cancel{\frac{1}{2}m} v_1^2 + \cancel{\frac{1}{2}m} v_2^2$$

So

(10.1) $$m\vec{v}_0 = m\vec{v}_1 + m\vec{v}_2 \qquad \text{(Eq 9.1)}$$

$$\vec{v}_0 \cdot \vec{v}_0 = (\vec{v}_1 + \vec{v}_2)\cdot(\vec{v}_1 + \vec{v}_2)$$

$$= \vec{v}_1^{\,2} + \vec{v}_2^{\,2} + 2\vec{v}_1\cdot\vec{v}_2$$

(10.2) $$v_0^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos(\theta + 35^\circ)$$

$$v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos(\theta + 35^\circ)$$

with (9.5

$$0 = 2v_1 v_2 \cos(\theta + 35^\circ)$$

$$\theta + 35^\circ = 90^\circ \implies \theta = 55^\circ$$

Then with Eqs (9.3) + (9.4)

(10.3) $$v_0 = v_1 \cos 35^\circ + v_2 \cos 55^\circ$$

(10.4) $$0 = v_1 \sin 35^\circ - v_2 \sin 55^\circ$$

(10.5) $$v_0 = v_1 \cos 35^\circ + v_1 \frac{\sin 35^\circ}{\sin 55^\circ} \cos 55^\circ$$

$$\frac{v_0}{\cos 35^\circ + \frac{\sin 35^\circ}{\sin 55^\circ}\cos 55^\circ} = v_1 \qquad v_2 = v_1 \frac{\sin 35^\circ}{\cos 35^\circ} = 1.14m$$

$$\frac{v_0 \sin 55^\circ}{\cos 35^\circ \sin 55^\circ + \sin 35 \cos 55^\circ} = v_1 \implies v_1 = v_0 \sin 55^\circ = 1.63m/$$

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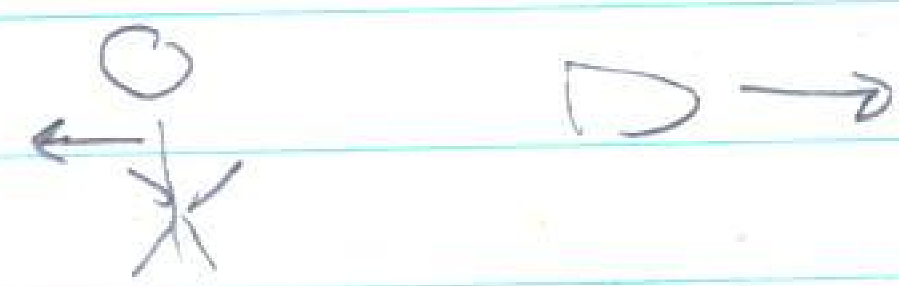
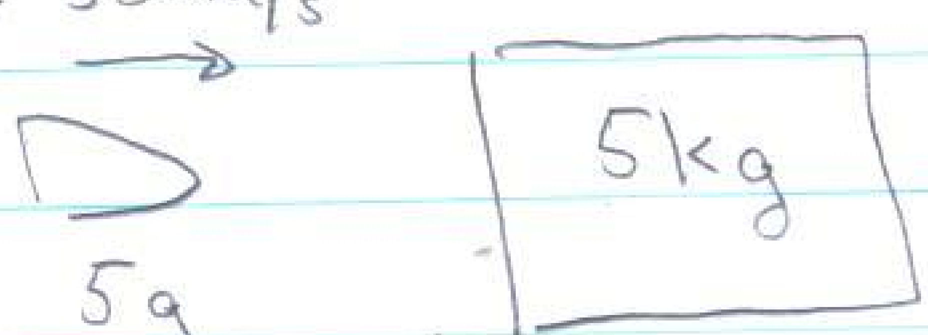


$p_{1i} + p_{2i} = 0$ $\vec{p}_{me} = -\vec{p}_{Bullet}$

Inelastic Collision:

Before

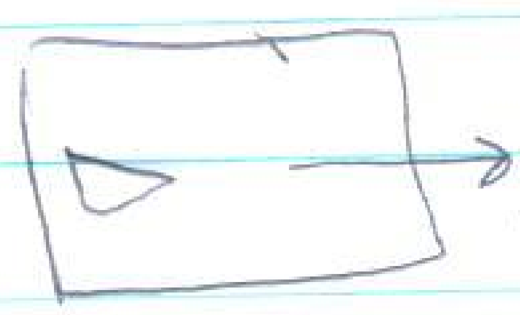
$v_0 = 500 m/s$



5kg

5g

After



v_f

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Find v_f:

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